

VII.

模型化

(加入年齡方式 未來豫想單位方式)

가

가

1. 가 47)

가. (Actuarial Assumptions)

(A) 가 (Demographic Assumptions):

· 加入年齡(entry age) = a (가 a 가)

· (normal retirement age; NRA) =

(normal pension age; NPA) = r (가)

· 가 (model membership):

(Stationary) 가 (Active members) 가 (Retired

members) ()

(, 가 l_a 가 l_r

가 (unit time period) 1)

47)

가

가 (Economic Assumptions):

- : $t = 0$
- 가 (salary growth rate) = $f(\text{ , } \text{ });$
- (promotional salary scale) $ss(x), a \text{ } x \text{ } r-1$
- (加入年齡 a . $ss(a) = ss(a-1) = 1$)
- (salary inflation rate) = $i_s, (1+i_s) = e, i_s > 0$
- 가 = s_0
- (Valuation interest rate) = $i_v, (1+i_v) = e, i_v > 0$

(, $i_s \text{ } i_v$. ' i_s & , i_v) > -1' 가

惹起 가

)

- 生存年金(Life Annuity)
- 期時給 . (Benefit Formula)

:

⇒ (Annual Pension) $AP = h(\text{年金給付發生率}) \times \text{加入年數} \times \text{退職時賃金}$

(C) 가 每年末 t ((discrete time approach));

(D) 每年初 , 連續時間接近法(continuous-time approach)

$r-1$ 1 가

.

$$(\ddot{a}_x^{(n)}) = \sum_{j=0}^{\infty} e^{-j} \times (l_{x+j}/l_x) = N_x / D_x$$

$${}_{n-1} | \ddot{a}_{x+1}^{(n)} = (l_{x+n}/l_{x+1}) \times e^{-(n-1)} \times \ddot{a}_{x+n}^{(n)} = (l_x/l_{x+1}) \times e^{-n} \times n | \ddot{a}_x^{(n)}$$

$$(\ddot{a}_{x:n}^{(n)}) = \sum_{j=0}^{n-1} e^{-j} \times (l_{x+j}/l_x) = (N_x - N_{x+n}) / D_x$$

$$\begin{aligned} n | \ddot{a}_x^{(n)} &= \sum_{j=n}^{\infty} e^{-j} \times (l_{x+j}/l_x) = \ddot{a}_x^{(n)} - \ddot{a}_{x:n}^{(n)} = (l_{x+n}/l_x) \times e^{-n} \times \ddot{a}_{x+n}^{(n)} \\ &= N_{x+n} / D_x \end{aligned}$$

$$\dot{S}_{x:n+1}^{(n)} = (l_{x+n}/l_{x+n+1}) \times e^{-n} \times (\dot{S}_{x:n}^{(n)} + 1)$$

$$(\dot{S}_{x:n}^{(n)}) = \ddot{a}_{x:n}^{(n)} / {}_n E_x^{(n)} = \sum_{j=0}^{n-1} e^{-(n-j)} \times (l_{x+j}/l_{x+n})$$

§ 7.3

$$PVB(t, x) = PVB(t, x; P) + PVB(t, x; F)$$

; t x 가 가 (t, x)

現價 (t, x) (past service)

가 , PVB(t, x; P), (t, x)

向後勤勞勤續(future service) 가

, PVB(t, x; F) .

§ 7.1 가 , x

= a, a+1, ..., r-1

PVB(t, x)

= Actuarial Present Value(APV) of future benefits for a member aged x, existing at time t, computed at time t

$$= h \times (r-a) \times s_0 \times e^{-(t+r-x)} \times_{r-x} | \ddot{a}_x^{(-)}$$

$$= \frac{h \times (x-a) \times s_0 \times e^{-(t+r-x)} \times_{r-x} | \ddot{a}_x^{(-)}}{}$$

(PVB(t, x; P); APV of Past service benefits)

$$+ \frac{h \times (r-x) \times s_0 \times e^{-(t+r-x)} \times_{r-x} | \ddot{a}_x^{(-)}}{}$$

(PVB(t, x; F); APV of Future service benefits)

, PVB(t, x), PVB(t, x; P), PVB(t, x; F)

$$PVB(t, x; P) = PVB(t, x) \times \frac{x-a}{r-a}$$

$$PVB(t, x; F) = PVB(t, x) \times \frac{r-x}{r-a}$$

; , PVB (pro rata fraction) .

3.

(; NC, AL)

算出

가

가 加入年齡方式(EAM) ,
 가 ,
 未來豫想單位方式(PUM)

補助積立方式 § 8.7

가. 加入年齡方式 (EAM)

加入年齡方式

EAM ‘ 가 가
 가 (,)
 가?’ -
 ‘ , 가 (,) 現價
 , 標準釀出金公式(standard contribution formula) 가
 (constant %) 平準
 積立 가 ;
 가 假想加入年齡(assumed
 entry age; e) e 가 假想가 (notional
 member) 가 가
 假想가 가 標準釀出金 ;

標準釀出金 標準負債 (,)가
 個別算定特性 가 (i.e. $r - e$) 가
 (projected final salary)
 未來發生給付算定特性 가 ;

가 가 e 가
 $e + 1$ $r - 1$ 標準負債
 AL(0) (PSL)가
 49). EAM $r - e$
 AL(0) 償却 () PSL
 (stable pace of funding)

標準釀出金(${}^E A NC$) 標準負債(${}^E A AL$)

(0)
 假想加入年齡(e) 가 (A2) 加入年齡(a) 加入年齡
 方式
 假想 가 가 (, $e = a$)

49) 가 e ()PSL = 0.
 , 가 e '()PSL > 0'
 가 .

(1)

t a 가 (salary function value) S(t, a) ,

$$S(t, a) = s_0 \times e^{-t} = S(t, x) \quad S(t) \quad \& \quad S(0, a) = S(0, x) = s_0$$

(가 (E) , 가

$$t \quad S(.) \quad)$$

$$S(t+1) = e \times S(t) \quad S(0) = s_0 \quad 50)$$

$${}^{EA}AP(t,a)$$

$${}^{EA}AP(t, a) = h \times (r-a) \times e^{-(r-a)t} \times S(t)$$

,

$${}^{EA}AP(t, x) = h \times (r-a) \times e^{-(r-x)t} \times S(t) \quad {}^{EA}H \times {}^{EA}PS(t, x),$$

x = a, a+1, ..., r-1.

$${}^{EA}H = h \times (r-a), \quad {}^{EA}PS(t, x) = e^{-(r-x)t} \times S(t) \quad 51).$$

(2)

(t, a) 가 가 r

$$50) \quad 1 \quad \text{(first-order recursive growth equation)}$$

function) (time-indexed dynamic growth model) 가 .

51) EAM (iii) 가 (i.e. r-a, years of the total (past & future) pensionable service)

$${}^{EA}H \quad , \quad {}^{EA}PS(t, x) \quad (t, x)$$

(projected final salary) . § 7.1 가 (C)

$$\text{年初} \quad {}^{EA}PS(t, x)^* = e^{-(r-x-1)t} \times$$

$$S(t) \text{ (i.e. 가 } r-1 \text{ 가)} \quad r-1$$

$$1 \quad \text{가} \quad \text{가} \quad {}^{EA}PS(t, x) = {}^{EA}PS(t, x)^* \times e .$$

(t, a) 現價 ${}^{EA}PVB(t, a)$

$${}^{EA}PVB(t, a) = {}^{EA}AP(t, a) \times {}_{r-a}| \ddot{a}_a^{(\cdot)}$$

$$, {}^{EA}PVB(t, x) = {}^{EA}AP(t, x) \times {}_{r-x}| \ddot{a}_x^{(\cdot)}, x = a, a+1, \dots, r-1.$$

(7.1) ${}^{EA}PVB(t, x)$ 가 § 7.2

$PVB(t, x)$

(3)

(t, a) 가 가 가 (i.e. r-a)

(t, a) 現價

${}^{EA}PVFS(t, a)$

$${}^{EA}PVFS(t, a) = S(t) \times \sum_{x=a}^{r-1} e^{-(x-a)} \times e^{-\delta(x-a)} \times 1_x/1_a$$

$$= S(t) \times \ddot{a}_{a:r-a}^{(\cdot)}$$

$$, {}^{EA}PVFS(t, x) = S(t) \times \ddot{a}_{x:r-x}^{(\cdot)}, x = a, a+1, \dots, r-1.$$

(4) - (i)

${}^{EA}PVFS(t, a)$ ${}^{EA}PVB(t, a)$ (

${}^{EA}PVFS(t, a)$ 가 ${}^{EA}PVB(t, a)$)

$${}^{EA}PVB(t, a) / {}^{EA}PVFS(t, a) = {}^{EA}H \times e^{-(r-a)} \times [{}_{r-a}| \ddot{a}_a^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)}],$$

independent of time t

; t a 가 (i.e.

r-a) 가 S(t)

(level fraction of current salary)

$$[{}^E PVB(t, x) / {}^E PVFS(t, x)] = {}^E H \times e^{-(r-x)t} \times [{}_{r-x} \ddot{a}_x^{(r-x)}] / \ddot{a}_{x:r-x}^{(r-x)}, x = a, a+1, \dots, r-1.$$

(5) - 假想가 標準釀出金

假想가 t 標準釀出金 ${}^E NC(t, a)$

$$\begin{aligned} {}^E NC(t, a) &= [{}^E PVB(t, a) / {}^E PVFS(t, a)] \times S(t) \\ &= {}^E PVB(t, a) / [{}^E PVFS(t, a) / S(t)] \\ &= {}^E PVB(t, a) / \ddot{a}_{a:r-a}^{(r-a)} \end{aligned} \quad \text{--- (7.1)}$$

$${}^E PVFNC(t, a) = {}^E PVB(t, a) \times \ddot{a}_{a:r-a}^{(r-a)} \quad ({}^E PVFNC(t, a) = {}^E NC(t, a) \times \ddot{a}_{a:r-a}^{(r-a)})$$

; $[{}^E PVB(t, a) / \ddot{a}_{a:r-a}^{(r-a)}]$ ${}^E PVB(t, a)$
(i.e. $r-a$) (uniformly

spreading-out over term $r-a$). , (t, a) $(r-a)$

標準釀出金 ${}^E NC(t, a)$
(discount function value) $e^{-(r-a)t}$ (t, a) 現價

(i.e. ${}^E NC(t, a) \times \ddot{a}_{a:r-a}^{(r-a)}$ r
 (t, a) 가 $PVB(t, a)$

§ 6.7.가. “釀出金積立原則” 收支相等 가

(t, a) (t, a)

${}^E AL(t, a) = 0$).

(7.2) 가 (i.e.

$a+1$ ($r-1$) 加入年齡 $NC(t, a) < NC(t, a+1) < \dots <$
 $NC(t, r-1)$ 가 , EAM (iii) &
 (iv) ${}^{EA}NC(t, a) = {}^{EA}NC(t, a+1) = \dots = {}^{EA}NC(t, r-1)$
 가 . 高 高標準釀出金
 假想加入年齡 가 가 假想가

個別加入年齡方式(Individual Entry Age
 method(IEAM) EAM 單一假想加
 入年齡 前提 가
 假想加入年齡 가 .
 高 高標準釀出金 가 ,

$$\begin{aligned}
 & {}^{EA}NC(t, a) \dots , \\
 & {}^{IEA}NC(t, x) = [{}^{IEA}PVB(t, x) / {}^{IEA}PVFS(t, x)] \times S(t) \\
 & = {}^{IEA}PVB(t, x) / \ddot{a}_{x:r-x}(\dots) , \quad x = a, a+1, \dots, r-1. \\
 & {}^{IEA}PVFNC(t, x) = {}^{IEA}PVB(t, x) \\
 & \quad ({}^{IEA}PVFNC(t, x) = {}^{IEA}NC(t, x) \times \ddot{a}_{a:r-a}(\dots)) \\
 & ; \quad \text{加入年齡(,)} \quad \text{收支相等}
 \end{aligned}$$

(7.3) § 6.7. . 先一時金方式(Initial Funding Method;
 IFM) 가 ${}^{IF}NC(t, a) = {}^{IF}PVB(t, a)$
 $= {}^{EA}PVB(t, a)$. (i.e. $r-a$) 現價
 Once-for-all payment
 EAM 大別

$$\begin{aligned}
{}^{EA}PVFNC(t, x) &= {}^{EA}NC(t, x) + {}^{EA}NC(t+1, x+1) \times e^{-1} \times l_{x+1}/l_x + \\
&\quad {}^{EA}NC(t+2, x+2) \times e^{-2} \times l_{x+2}/l_x + \dots + \\
&\quad {}^{EA}NC(t+r-x-1, r-1) \times e^{-1} \times l_{r-1}/l_r \\
&= {}^{EA}NC(t, x) \times \ddot{a}_{x:r-x}^{(\cdot)} \quad (= {}^{EA}NC(t, a) \times \ddot{a}_{x:r-x}^{(\cdot)}) \\
({}^{EA}NC(t+j, x+j) &= {}^{EA}NC(t, x) \times e^{-j}, \quad 1 \leq j \leq r-x-1 \quad \& \quad {}^{EA}NC(t, a) \\
&= {}^{EA}NC(t, x))
\end{aligned}$$

$$\begin{aligned}
&{}^{EA}AL(t, x) \\
&= {}^{EA}PVB(t, x) - {}^{EA}PVFNC(t, x) \\
&= [{}^{EA}AP(t, x) \times {}_{r-x} | \ddot{a}_x^{(\cdot)}] - {}^{EA}NC(t, a) \times \ddot{a}_{x:r-x}^{(\cdot)} \\
&= {}^{EA}H \times FS(t, x) \times {}_{r-x} | \ddot{a}_x^{(\cdot)} - {}^{EA}H \times S(t) \times e^{-(r-a)} \times [{}_{r-a} | \ddot{a}_a^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)}] \\
&\quad \times \ddot{a}_{x:r-x}^{(\cdot)} \\
&= \{ {}^{EA}H \times S(t) \times e^{-(r-x)} \times {}_{r-x} | \ddot{a}_x^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)} \} \times \\
&\quad \{ \ddot{a}_{a:r-a}^{(\cdot)} - e^{-(x-a)} \times [{}_{r-a} | \ddot{a}_a^{(\cdot)} / {}_{r-x} | \ddot{a}_x^{(\cdot)}] \times \ddot{a}_{x:r-x}^{(\cdot)} \} \\
&= \{ {}^{EA}H \times S(t) \times e^{-(r-x)} \times {}_{r-x} | \ddot{a}_x^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)} \} \times \{ \ddot{a}_{a:r-a}^{(\cdot)} - e^{-(\cdot)(x-a)} \times
\end{aligned}$$

52) IV 가 ${}^{EA}AL(t, x)$ 가
 ${}^{EA}NC(t, x)$ 가 , ${}^{EA}AL(t, x)$
 ${}^{EA}NC(t, x)$ 가 .
(continuous-time approach)
(0)
notation .
notation : t
(i.e. t-1) & t+0 (i.e. § 7.1 가
(D) t 가
). ${}^{EA}AL(t, x)$, ${}^{EA}NC(t+0, x)$. notation

$$\begin{aligned}
& (l_x/l_a) \times \ddot{a}_{x:r-x}^{(\cdot)} \\
& (\quad | \ddot{a}_a^{(\cdot)} / | \ddot{a}_x^{(\cdot)} = e^{- (x-a)} \times l_x / l_a) \\
& = \{ {}^E A H \times S(t) \times e^{- (r-x)} \times | \ddot{a}_x^{(\cdot)} \} \times \{ \ddot{a}_{a:x-a}^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)} \} \\
& (\quad \{ \ddot{a}_{a:r-a}^{(\cdot)} - e^{- (\cdot)(x-a)} \times (l_x/l_a) \times \ddot{a}_{x:r-x}^{(\cdot)} \} = \ddot{a}_{a:x-a}^{(\cdot)}) \\
& = {}^E A PVB(t, x) \times [\ddot{a}_{a:x-a}^{(\cdot)} / \ddot{a}_{a:r-a}^{(\cdot)}] \quad \text{--- (7.3)}
\end{aligned}$$

; § 7.1 가 가 ,
(t, a)가 ${}^E A AL(t, a) = 0$ 가 가 가 標準負債
가 (t+r-a, r) 年金財
源 (i.e. ${}^E A AL(t+r-a, r) =$
PVB(t+r-a, r)).

$$, \quad (7.3) \quad {}^E A AL(t, a) \quad {}^E A NC(t, a)$$

;

$$\begin{aligned}
& {}^E A AL(t, x) \\
& = {}^E A NC(t, a) \times e^{- (x-a)} \times | \ddot{a}_x^{(\cdot)} / | \ddot{a}_a^{(\cdot)} \times \ddot{a}_{a:x-a}^{(\cdot)} \\
& = {}^E A NC(t, a) \times \ddot{a}_{a:x-a}^{(\cdot)} \times e^{- (\cdot)(x-a)} \times (l_a/l_x) \\
& = {}^E A NC(t, a) \times \dot{s}_{a:x-a}^{(\cdot)} \quad (\quad \S 7.2 \quad) \quad \text{--- (7.4)}
\end{aligned}$$

; § 7.1 가 가 ,
가 (t-x+a, a)가 (t, x)
가 (x-a) 標準釀出金 $\{ {}^E A NC(t-x+a, a)$
, ${}^E A NC(t-x+a+1, a)$, ... , ${}^E A NC(t-2, a)$, ${}^E A NC(t-1, a)$ } = ${}^E A NC(t, a) \times$

$$e^{-(x-a)}, \dots, {}^{EA}NC(t, a) \times e^{-2}, {}^{EA}NC(t, a) \times e^{-1} \},$$

年始給 標準釀出金
累積終價(actuarial accumulated value of the normal costs)

(7.4)

$$(7.3) \quad \text{(Prospective Method)} \quad (7.4)$$

(Retrospective Method)

가

$\cdot (t, x)$
標準負債, ${}^{EA}AL(t, x), r-x$
,
標準釀出金
(t, x)
가
가 (A2)
(t+r-x, r)
. x = r, r+1, r+2, ...
,

$$\begin{aligned}
 & {}^{EA}AL(t, x) \\
 &= {}^{EA}PVB(t, x) \\
 &= \frac{{}^{EA}AP(t+r-x, r)}{r} \times \ddot{a}_x^{(r)} \\
 & \quad \left(\text{t} \quad \text{x} \quad \text{가} \quad \text{r} \quad \text{(t+r-x)} \right) \\
 &= {}^{EA}H \times \frac{S(t+r-x)}{r} \times \ddot{a}_x^{(r)} \\
 & \quad \uparrow \left(\text{t} \quad \text{x} \quad \text{가} \quad \text{r} \quad \text{(t+r-x)} \right) \\
 &= {}^{EA}H \times e^{-(r-x)} \times S(t) \times \ddot{a}_x^{(r)} \quad \text{--- (7.5)}
 \end{aligned}$$

, §7.1 가 (F) (t, x)

$${}^{EA}EB(t, x)$$

$${}^{EA}EB(t, x) = {}^{EA}AP(t+r-x, r) \quad \text{--- (7.6)}$$

, (7.3) & (7.5) t 標準負債 ${}^{EA}AL(t)$

$${}^{EA}AL(t)$$

$$= \sum_{x=a} [l_x \times {}^{EA}AL(t, x)]$$

$$= \left\{ \sum_{x=a}^{r-1} [l_x \times {}^{EA}H \times e^{-(r-x)} \times S(t) \times {}_{r-x} | \ddot{a}_x^{()} \times \ddot{a}_{a:x-a}^{()} / \ddot{a}_{a:r-a}^{()}] \right\}$$

$$+ \sum_{x=r} [l_x \times {}^{EA}H \times e^{-(r-x)} \times S(t) \times \ddot{a}_x^{()}] \}$$

$$= \frac{1}{W(t)} \times \left\{ \sum_{x=a}^{r-1} [l_x \times {}^{EA}H \times e^{-(r-x)} \times S(t) \times {}_{r-x} | \ddot{a}_x^{()} \times \ddot{a}_{a:x-a}^{()} / \ddot{a}_{a:r-a}^{()}] \right\}$$

$$+ \sum_{x=r} [l_x \times {}^{EA}H \times e^{-(r-x)} \times S(t) \times \ddot{a}_x^{()}] \} \times W(t)$$

$$= \left[\left\{ \sum_{x=a}^{r-1} [l_x \times {}^{EA}H \times e^{-(r-x)} \times {}_{r-x} | \ddot{a}_x^{()} \times \ddot{a}_{a:x-a}^{()} / \ddot{a}_{a:r-a}^{()}] \right\} \right.$$

$$\left. + \sum_{x=r} [l_x \times {}^{EA}H \times e^{-(r-x)} \times \ddot{a}_x^{()}] \right\} / \sum_{x=a}^{r-1} l_x \times W(t)$$

$${}^{EA}a\% \times W(t) \quad \text{--- (7.7)}$$

; 標準釀出金 (%) 標準

負債

(7.5) (7.2) EAM

:

(i.e. $t=0$) x 가 標準釀出金 ${}^{EA}NC(0, x)$
 $(x-a)$ 初期標準負債 ${}^{EA}AL(0, x)$,
 $x = a+1, a+2, \dots, r-1$ (${}^{EA}AL(0, a) = 0$),

(7.1),

$${}^{EA}NC(0, x) = {}^{EA}NC(0, a) = {}^{EA}H \times S(0) \times e^{-(r-a)} \times [{}_{r-a} \ddot{a}_a^{(r-a)} / \ddot{a}_{a:r-a}^{(r-a)}]$$

(7.2) 0 標準釀出金

$$\begin{aligned} & {}^{EA}NC(0) \\ &= \left[\sum_{x=a}^{r-1} l_x \right] \times {}^{EA}NC(0, a) \\ &= [{}^{EA}PVB(t, a) / {}^{EA}PVFS(t, a)] \times \sum_{x=a}^{r-1} [l_x \times S(0)] \\ &= {}^{EA}c\% \times W(0) \end{aligned} \quad \text{--- (7.8)}$$

; (7.2)

$$(7.7) \quad t=0 \quad {}^{EA}AL(0)$$

(,)가 § 7.1 가 (A1)

EAM

(i.e. $x-a$) EAM 負債
 債 . x 가

$$\text{가 } PSL(x) \text{ , } a \text{ } 0+a-x$$

가 0 x 가

$$\text{負債 } (7.3) \text{ (i.e. } t=0) \text{ } S(0)$$

$$\text{ , } S(0) = s_0 \times e^{-(x-a)}$$

$PSL(0, x)$

$$= \{ {}^{EA}H \times S(0) \times e^{-(r-x)} \times {}_{r-x} \ddot{a}_x^{(r-x)} \} \times [\ddot{a}_{a:x-a}^{(r-x)} / \ddot{a}_{a:r-a}^{(r-x)}]$$

$$= e^{-r} \times W(t), \quad W(0) \quad .$$

$$\begin{aligned} {}^{EA}NC(t+1) &= {}^{EA}c\% \times W(t+1) \\ &= {}^{EA}c\% \times e^{-r} \times W(t) \\ &= e^{-r} \times {}^{EA}NC(t), \quad {}^{EA}NC(0) \quad \text{--- (7.9)} \end{aligned}$$

; , ${}^{EA}NC(0)$ 가 (7.7) ${}^{EA}NC(t)$ 가 , 標準釀出金

(9) - ${}^{EA}AL$ 再歸等式(recursive equation)

$$\begin{aligned} \cdot {}^{EA}AL(t) &= {}^{EA}AL(t+1) \quad \text{標準負債} \\ &\quad \text{(projection)} \quad (, \quad \text{PSL} \quad \text{(7.7)} \end{aligned}$$

(7.11) ${}^{EA}AL$ PSL EAM (補助積立方式)

\cdot EAM (,)가

\cdot §7.1 가 (F)

가 $x = a, a+1, \dots, r-1,$ (7.1) & (7.4)

$$\begin{aligned} &e^{-rx} \times [l_x \times {}^{EA}AL(t, x) + l_x \times {}^{EA}NC(t, x)] \\ &= e^{-rx} \times \{ [l_x \times {}^{EA}NC(t, x) \times \dot{s}_{a:x-a}^{(r)}] + [l_x \times {}^{EA}NC(t, x)] \} \\ &= e^{-rx} \times l_x \times {}^{EA}NC(t, x) \times [\dot{s}_{a:x-a}^{(r)} + 1] \\ &= l_x \times (l_{x+1}/l_x) \times {}^{EA}NC(t, x) \times e^{-rx} \times [\dot{s}_{a:x-a}^{(r)} \times (l_x/l_{x+1}) \times e^{-(x-a)} + (l_x/l_{x+1}) \times e^{-(x-a)}] \end{aligned}$$

$$\begin{aligned}
&= l_{x+1} \times {}^{EA}NC(t+1, x+1) \times [\dot{S}_{a:x+1-a}^{(r)}] \quad (\S 7.2) \\
&= l_{x+1} \times {}^{EA}AL(t+1, x+1) \quad \text{--- (7.10)}
\end{aligned}$$

$$p_x \times {}^{EA}AL(t+1, x+1) = e \times [{}^{EA}AL(t, x) + {}^{EA}NC(t, x)], \quad p_x = l_{x+1}/l_x$$

가 , $x = r, r+1, r+2, \dots$, (7.5) & (7.6)

$$\begin{aligned}
&e \times [l_x \times {}^{EA}AL(t, x) - l_x \times {}^{EA}EB(t, x)] \\
&= e \times l_x \times [{}^{EA}AP(t+r-x, r) \times \ddot{a}_x^{(r)} - {}^{EA}AP(t+r-x, r)] \\
&= l_x \times (l_{x+1}/l_x) \times {}^{EA}AP(t+r-x, r) \times [\ddot{a}_x^{(r)} \times (l_x/l_{x+1}) \times e - (l_x/l_{x+1}) \times e] \\
&= l_{x+1} \times {}^{EA}AP(t+r-x, r) \times \ddot{a}_{x+1}^{(r)} \quad (\S 7.2) \\
&= l_{x+1} \times {}^{EA}AL(t+1, x+1) \quad \text{--- (7.11)}
\end{aligned}$$

$$p_x \times {}^{EA}AL(t+1, x+1) = e \times [{}^{EA}AL(t, x) - {}^{EA}EB(t, x)], \quad p_x = l_{x+1}/l_x$$

, (,)가 , $x = a, a+1, \dots, r, r+1, \dots$, (7.10)
& (7.11) (7.12) 가 :

$$e \times \left\{ \sum_{x=a}^{\infty} [l_x \times {}^{EA}AL(t, x)] + \sum_{x=a}^{r-1} [l_x \times {}^{EA}NC(t, x)] - \sum_{x=r}^{\infty} [l_x \times {}^{EA}EB(t, x)] \right\} (= e \times \{ {}^{EA}AL(t) + {}^{EA}NC(t) - {}^{EA}EB(t) \})$$

$$= \sum_{x=a}^{\infty} [l_{x+1} \times {}^{EA}AL(t+1, x+1)]$$

$$= {}^{EA}AL(t+1)$$

$${}^{EA}AL(t+1) = e \times \{ {}^{EA}AL(t) + {}^{EA}NC(t) - {}^{EA}B(t) \}, \quad {}^{EA}AL(0) \quad \text{--- (7.12)}$$

; ${}^{EA}AL(0)$ PSL
 EAM (, (7.10))

, 가 EAM 標準負債 (7.12)

$\{{}^{EA}AL(t): t=0, 1, 2, \dots\}$

標準負債

標準釀出金

(7.10) 가 : , (7.10)

$${}^{EA}AL(t) = {}^{EA}a\% \times W(t)$$

$${}^{EA}AL(t+1) = {}^{EA}a\% \times W(t+1)$$

$$= {}^{EA}a\% \times \sum_{x=a}^{r-1} [l_x \times S(t+1)]$$

$$= {}^{EA}a\% \times e \times \sum_{x=a}^{r-1} [l_x \times S(t)]$$

$$= e \times {}^{EA}AL(t), \quad AL(0) \quad \dots (7.13)$$

; 標準負債

· 未來豫想單位方式 (PUM)

§ 7.1 가 (A) (G) 加入年齡方式 ,

加入年齡方式

未來豫想單位方式

:

PUM 가 (,) 가? 가 (,)
 - ' ' 가 (,)
 現價 .
 가 가 (§7.1 가 1)
 (single premium basis) 標準釀出金
 , 가
 가 既發生標準負債(accrued actuarial liability) 가 ;

EAM 假想가 標準釀出金 PUM
 가 (actual age) .
 EAM PUM ()PSL
 (AL(0) = 0);

標準釀出金 負債 ()
 ,)가 個別算定特性 가
 (actual service period) 既發生給付算定特性 (,)
 가 100%
 完全積立) 가

(Long-term funding requirement) 가 .

標準釀出金(^PU NC) 負債(^PU AL)

(0)

EAM 假想가 가

(1)

t a 가 (salary function value) S(t, a) ,

$$S(t, a) = s_0 \times e^{-t} = S(t, x) \quad S(t)$$

(가 (C) , 가

$$S(t+1) = e \times S(t) \quad S(0) = s_0$$

$${}^{EA}AP(t,a)$$

$${}^{PU}AP(t, a) = h \times (r-a) \times e^{-(r-a)t} \times S(t) (= {}^{EA}AP(t, a))$$

${}^{PU}AP(t, x) = h \times (r-x) \times e^{-(r-x)t} \times S(t) \quad {}^{PU}H(x) \times {}^{PU}PS((t), x)$
(${}^{EA}AP(t, x), x = a+1, a+2, \dots, r-1$).

$${}^{PU}H(x) = h \times (r-x) \quad ({}^{EA}H, \quad {}^{PU}H(a) = {}^{EA}H), \quad {}^{PU}PS(t, x) = e^{-(r-x)t} \times S(t) (= {}^{EA}PS(t, x)) \quad 53).$$

53) ${}^{EA}H$ ${}^{PU}H(x)$ EAM PUM 가 (i.e. r-a) , EAM ${}^{EA}H$, 가 (i.e. x-a) t 時點以前 100% (

(2) - (i)
 ${}^{PU}AP(t, x)$ 가 (i.e. $r-x$)
 . 가 (i.e. 1)
 ${}^{PU}(t, x)$

${}^{PU}AP(t, x) / (r-x)$
 $= h \times {}^{PU}PS(t, x)$
 ${}^{PU}(t, x)$
 ; (t, x) 가 가 (t, x) (t+1, x+1)
 (projected unit benefit) .

(3) - 標準釀出金

§ 7.2 , (t, x) 가 가 (t, x)
 (t, x) 現價

${}^{PU}PVB(t, x; F)$
 ${}^{PU}PVB(t, x; F)$
 $= {}^{PU}AP(t, x) \times {}_{r-x} | \ddot{a}_x^{()}$, $x = a, a+1, \dots, r-1$
 (= $PVB(t, x; F)$, § 7.2)

가 (i.e. $r-x$) $r-x$ (t,
 x) (t+1, x+1)

(iii))
 (i.e. $r-x$) PUM ${}^{PU}H(x)$
 . , ${}^{EA}PS(t, x) = {}^{PU}PS(t, x)$ 가
 EAM PUM 가 (projected final salary)

(t, x) 現價 ${}^{PU}PVB(t, x; F)/(r-x)$.
 (t, x) (t+1, x+1) 標準釀出金

$$\begin{aligned}
 & {}^{PU}NC(t, x) \\
 &= {}^{PU}PVB(t, x; F) / (r-x) \\
 &= \frac{{}^{PU}PVB(t, x)}{(r-a)} \\
 &\quad (= {}^{EA}PVB(t, x) = PVB(t, x), \text{ § 7.2 }) \\
 &= {}^{PU}(t, x) \times {}_{r-x}| \ddot{a}_x^{()} \\
 &= \frac{h \times e^{-(t-x)} \times {}_{r-x}| \ddot{a}_x^{()}}{(EAM \text{ 가 } S(t), \%)} \times S(t), \quad x = a, a+1, \dots, r-1. \quad \text{--- (7.14)}
 \end{aligned}$$

; (t, x) 가 가 勤續 r
 (i.e. ${}^{PU}AP(t+r-x, r) \times \ddot{a}_r^{()}$) (t, x)
 (t+1, x+1) .

(7.6) 既發生標準負債(accrued actuarial liability) 가
 EAM ${}^{EA}PVFS(t, a)$

(4) - (iii)
 t 標準釀出金 ${}^{EA}NC(t)$

$${}^{PU}NC(t) = \sum_{x=a}^{r-1} [l_x \times {}^{PU}NC(t, x)]$$

$$\begin{aligned}
&= \left[\frac{1}{W(t)} \times \left\{ \sum_{x=a}^{r-1} [h \times e^{(r-x)} \times r-x \ddot{a}_x^{()} \times l_x \times S(t)] \right\} \right] \\
&\quad \times W(t) \\
&= \left[\left\{ \sum_{x=a}^{r-1} [h \times e^{(r-x)} \times r-x \ddot{a}_x^{()} \times l_x] \right\} / \left\{ \sum_{x=a}^{r-1} l_x \right\} \right] \times W(t) \\
&= {}^{PU}c\% \times W(t) \qquad \text{--- (7.15)}
\end{aligned}$$

; EAM ${}^{EA}NC(t)$ t 가
(total pensionable payroll; $W(t)$) (%)

(7.2) ${}^{EA}c$ 假想가
, ${}^{PU}c$ 가

(5) - 負債

. PUM 既發生給付算定特性 가
負債 既發生標準負債(Accrued Actuarial Liability) - §
7.1 가 , 가 負債
評價日直前 標準釀出金 累積終價 .
, $(t+x-a, x)$ 가 負債 ${}^{PU}AL(t+x-a, x), x = a,$
 $a+1, a+2, \dots, r-1,$ §7.1 (A1)가 (t, a) l_a 가
가 $(t+x-a, x)$ l_x 標準釀出金
가(accumulated value of the normal costs)

$$\begin{aligned}
&l_x \times AL(t+x-a, x) \\
&= \sum_{j=0}^{x-a-1} l_{a+j} \times NC(t+j, a+j) \times e^{-(x-a-j)}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{j=0}^{x-a-1} l_{a+j} \times h \times e^{(r-a-j)} \times r_{r-a-j} | \ddot{a}_{a+j}^{(r)} \times \underline{S(t+j)} \times e^{-(x-a-j)} \\
&\hspace{20em} (= S(t) \times e^{-j}) \\
&= h \times S(t) \times \left\{ \sum_{j=0}^{x-a-1} l_{a+j} \times e^{(r-a)} \times e^{-(r-a-j)} \times (l_r/l_{a+j}) \times \ddot{a}_r^{(r)} \times e^{-(x-a-j)} \right\} \\
&\quad (\S 7.2 \quad) \\
&= h \times S(t) \times e^{(r-a)} \times \left\{ \sum_{j=0}^{x-a-1} e^{(r-a)} \times e^{-(r-x)} \times (l_r/l_k) \times l_r \times \ddot{a}_r^{(r)} \right\} \\
&= h \times \underline{S(t+x-a)} \times e^{-(t-x)} \times l_k \times \underline{(x-a)} \times r_{r-x} | \ddot{a}_x^{(r)} \hspace{10em} \text{--- (7.16)} \\
&\hspace{15em} (\quad \text{가} \quad) \\
&\hspace{10em} ((t+x-a, x) \quad \text{가} \quad , S(t+x-a), \quad r \\
&\hspace{15em})
\end{aligned}$$

; (t, a)가 **累積終價** 가 0
0 가 가 가 가가 가
(t+r-a, r) (i.e. $h \times S(t+r-a) \times l_r \times (r-a)$
 $\times \ddot{a}_r^{(r)}$) 100% **完全積立** PUM

, (7.16) (t, x) 가 **標準負債**
 ${}^{\text{PU}}\text{AL}(t, x), x = a, a+1, a+2, \dots, r-1,$

$$\begin{aligned}
&{}^{\text{PU}}\text{AL}(t, x) \\
&= h \times S(t) \times e^{(r-x)} \times (x-a) \times r_{r-x} | \ddot{a}_x^{(r)} \\
&= {}^{\text{PU}}\text{PVB}(t, x; P) \\
&= h \times S(t) \times e^{(r-x)} \times (r-a) \times r_{r-x} | \ddot{a}_x^{(r)} \times [(x-a)/(r-a)] \hspace{10em} \text{--- (7.17)}
\end{aligned}$$

$$= {}^{\text{PU}}\text{PVB}(t, x) \times \frac{(x-a)}{(r-a)}$$

; , 標準負債 PVB (i.e. $(x-a)/(r-a)$) ,
 EAM 標準負債가 (7.3) PVB (i.e.
 $\ddot{a}_{a:x-a}^{(r-a)} / \ddot{a}_{a:r-a}^{(r-a)}$) .

(7.7) (7.2) EAM 標準負債算定
 . PUM (7.16)
 . , (7.16)

· (t, x) 負債, ${}^{\text{PU}}\text{AL}(t, x)$, $r < x$
 , EAM . , $x = r, r+1, r+2, \dots$,

$$\begin{aligned} & {}^{\text{PU}}\text{AL}(t, x) \\ &= {}^{\text{PU}}\text{PVB}(t, x) \\ &= h \times (r-a) \times \frac{S(t+r-x)}{(t+r-x)} \times \ddot{a}_x^{(r-a)} (= {}^{\text{EA}}\text{AL}(t, x)) \end{aligned}$$

$$= h \times (r-a) \times e^{-(r-x)} \times S(t) \times \ddot{a}_x^{(r)} \quad \text{--- (7.18)}$$

, § 7.1 가 (F) (t, x)
 ${}^{EA}EB(t, x)$

$${}^{PU}EB(t, x) = {}^{PU}AP(t+r-x, r) (= {}^{EA}EB(t, x)) \quad \text{--- (7.19)}$$

, (7.16) & (7.17) t 負債 ${}^{EA}AL(t)$

${}^{PU}AL(t)$

$$\begin{aligned} &= \sum_{x=a}^{r-1} [l_x \times {}^{PU}AL(t, x)] \\ &= \left\{ \sum_{x=a}^{r-1} [l_x \times h \times S(t) \times e^{-(r-x)} \times (x-a) \times r_{r-x} | \ddot{a}_x^{(r)}] \right. \\ &\quad \left. + \sum_{x=r} [l_x \times h \times (r-a) \times e^{-(r-x)} \times S(t) \times \ddot{a}_x^{(r)}] \right\} \\ &= \left[\left\{ \sum_{x=a}^{r-1} [l_x \times h \times e^{-(r-x)} \times (x-a) \times r_{r-x} | \ddot{a}_x^{(r)}] \right. \right. \\ &\quad \left. \left. + \sum_{x=r} [l_x \times h \times (r-a) \times e^{-(r-x)} \times \ddot{a}_x^{(r)}] \right\} / \sum_{x=a}^{r-1} l_x \right] \times W(t) \\ &{}^{PU}a\% \times W(t) \quad \text{--- (7.20)} \end{aligned}$$

; 標準釀出金 (%)

負債 .

(7.8) EAM PUM 가 標準負債

()PSL

. , PUM EAM

100%

, 가

EAM

(Initial Funding Method)

가 .

가 Mcleish & Steward(1987)

既發生定義方式(Defined Accrued Benefit Method); Marshall & Reeve(1993)

, (long - term)
(short - term)

(6) - ^{PU}NC 再歸成長等式(recursive growth equation)

EAM PUM 標準釀出金 再歸等式

: ,

(7.15)

$${}^{PU}NC(t) = {}^{PU}c\% \times W(t) \quad \&$$

$$W(t) = \sum_{x=a}^{t-1} [l_x \times S(t)], \quad t = 0, 1, 2, \dots$$

$$\begin{aligned} {}^{PU}NC(t+1) &= {}^{PU}c\% \times W(t+1) \\ &= {}^{PU}c\% \times e \times W(t) \\ &= e \times {}^{PU}NC(t), \quad {}^{PU}NC(0) \quad \dots (7.21) \end{aligned}$$

; (7.21)

$\{ {}^{PU}NC(t): t = 0, 1, 2, \dots \}$ 가

標準釀出金

(7) - ${}^{PU}AL$ 再歸等式(recursive equation)

· EAM ${}^{PU}AL(t) \quad {}^{PU}AL(t+1)$

負債 (projection)

(,)가

가 $x = a, a+1, \dots, r-1,$ (7.14) & (7.17)

$$\begin{aligned}
 & e \times [l_x \times {}^{PU}AL(t, x) + l_x \times {}^{PU}NC(t, x)] \\
 = & e \times l_x \times h \times S(t) \times e^{-(r-x)} \times (x+1-a) \times {}_{r-x} | \ddot{a}_x^{(-)} \\
 & \text{((7.11) \& (7.14))} \\
 = & l_{x+1} \times h \times S(t) \times e^{-(r-x)} \times (x+1-a) \times \left[\frac{e^{-x} \times (l_x/l_{x+1}) \times {}_{r-x} | \ddot{a}_x^{(-)}}{e^{-r-(x+1)} \ddot{a}_{x+1}^{(-)}}, \S 7.2 \right] \\
 = & l_{x+1} \times {}^{PU}AL(t+1, x+1) \quad \text{--- (7.22)}
 \end{aligned}$$

$$p_x \times {}^{PU}AL(t+1, x+1) = e \times [{}^{PU}AL(t, x) + {}^{PU}NC(t, x)], \quad p_x = l_{x+1}/l_x$$

가 , $x = r, r+1, r+2, \dots,$ (7.17) & (7.19)

$$\begin{aligned}
 & e \times [l_x \times {}^{PU}AL(t, x) - l_x \times {}^{PU}EB(t, x)] \\
 = & e \times l_x \times [{}^{PU}AP(t+r-x, r) \times \ddot{a}_x^{(-)} - {}^{PU}AP(t+r-x, r)]
 \end{aligned}$$

$$\begin{aligned}
&= l_x \times (l_{x+1}/l_x) \times {}^P U AP(t+r-x, r) \times [\ddot{a}_x^{(1)} \times (l_x/l_{x+1}) \times e^{-r} - (l_x/l_{x+1}) \times e^{-r}] \\
&= l_{x+1} \times {}^P U AP(t+r-x, r) \times \ddot{a}_{x+1}^{(1)} \quad (\text{§ 7.2}) \\
&= l_{x+1} \times {}^P U AL(t+1, x+1) \quad \dots (7.23)
\end{aligned}$$

$$p_x \times {}^P U AL(t+1, x+1) = e^{-r} \times [{}^P U AL(t, x) - {}^P U EB(t, x)], \quad p_x = l_{x+1}/l_x$$

, EAM (,)가 , x = a, a+1, ..., r, r+1, ..., (7.22) & (7.23) (7.24)

가 :

$$\begin{aligned}
&e^{-r} \times \left\{ \sum_{x=a}^r [l_x \times {}^P U AL(t, x)] + \sum_{x=a}^{r-1} [l_x \times {}^P U NC(t, x)] - \sum_{x=r}^r [l_x \times {}^P U EB(t, x)] \right\} \\
&= e^{-r} \times \{ {}^E A AL(t) + {}^E A NC(t) - {}^E A EB(t) \} \\
&= \sum_{x=a}^r l_{x+1} \times {}^P U AL(t+1, x+1) \\
&= {}^P U AL(t+1)
\end{aligned}$$

$${}^P U AL(t+1) = e^{-r} \times \{ {}^P U AL(t) + {}^P U NC(t) - {}^P U EB(t) \}, \quad AL(0) \quad \dots (7.24)$$

EAM

負債

標準釀出金

(7.23) : , (7.22)

$${}^P U AL(t) = {}^P U a\% \times W(t)$$

$${}^P U AL(t+1) = {}^P U a\% \times W(t+1)$$

$$\begin{aligned}
&= {}^{PU}a\% \times \sum_{x=a}^{r-1} [l_x \times S(t+1)] \\
&= {}^{PU}a\% \times e \times \sum_{x=a}^{r-1} [l_x \times S(t)] \\
&= e \times {}^{PU}AL(t), \quad AL(0) \quad \text{--- (7.25)}
\end{aligned}$$

; 負債

EAM PUM

\	EAM	PUM
가	가 가 (i.e. e = a)	가 (i.e. e = x)
	r - a	r - x
NC	r - a (i.e.)	1 (i.e. (x+1) - x)
NC	PVB(t, a)/PVFS(t, a)	AP(t, x) / (r - x)
PVB AL	$\ddot{a}_{a:x-a}^{(r-a)} / \ddot{a}_{a:r-a}^{(r-a)}$	(x - a) / (r - a)
100%	PVB(t, r) = AL(t, r) if $x=r, \ddot{a}_{a:x-a}^{(r-a)} / \ddot{a}_{a:r-a}^{(r-a)}=1$	PVB(t, r) = AL(t, r) if $x=r, (x-a)/(r-a) = 1$
() PSL	PSL(0, x) = 0 ()	PSL(0, x) = 0

(7.1) : 加入年齡方式(EAM) 未來豫想單位方式(PUM)